Analytical Hierarchy Process as a Decision-Making Model

Agerti GALO

Lector in University "Ismail Qemali", Department of Finance, PhD candidate at Tirana University Finance Department

Abstract

When the problems we face are complex and affect each other, then the decision making process is more difficult. In most cases we apply established policies or choices without knowing which the best choice is. To make appropriate decisions that can solve the problems encountered should be analyzed very well the reasons that create problems and their reciprocal influence. AHP helps the decision of the people who will decide the problem by taking a hierarchical structure evaluation, opinions, experiences and all information about this problem. This flexible structure enables analytical feelings and instincts to organize and align with a shape that resembles human logic. Thus this analytical flexible structure, allowing to adjust the paper instead of the mind, gives people the opportunity to intervene in the most difficult problems and complex.

Keywords: decision-making, analytical hierarchy process, model

Introduction

The process of hierarchical analysis (AHP-Analytic Hierarchy Process) fell for the first time as an idea in 1968 by Mazers and Alpert, and in 1977 by Saaty was processed and turned into a model that can be used for decision making solving problems (Kaan Yaralıoğlu, 2006). AHP enables forecasting and decision-making by creating a hierarchical flexible structure about the problem that will be decided. The method is based on comparisons made by the importance of the twin values of these elements or elements that influence decision-making. The comparison is done using a previously defined table on the hierarchical structure formed about a problem.

Research methods

Banks or various institutions have a constant need for decision-makings. This need for decision-makings in most cases is immediate and in most others is also a *fast end solution* in decision. The accuracy of decision making is as much important as the speed of decision making. In this article is expressed the synthesis of a number of articles and studies on ways of decision making. Therefore it has reached the conclusion that the process of hierarchical analysis is a method which not only provides rapid feedback but also a method of easily usable by many levels of non-high qualified staff.

Methodology

The process of hierarchical analysis model is easily usable, highly flexible due to social and economic behavior changes and quite quick in decision-making. Creation of this model is quite easy and goes through stages aggravated and evaluative which mimic human logic and coherent based on estimates. The core of this model is the use of the matrix which is based on comparing two levels starting from basic to higher levels. Explaining the scheme used in this model and tracking model levels is essential for the final decision of this model.

Creating hierarchies

Created hierarchies can be both structural and functional types. Structural hierarchies are formed as a result of the process of deciding upon the class of elements, placing the highest level to the lowest, considering features such as age, color, size, elements that are inside the structure. Hierarchical structures mimic the human brain solving problems systemically while facing them.

While functional hierarchies are created by dividing complex system into smaller parts and simple, given the links under each - other. These elements are related to the problem generally fall under the criteria defined class, they are divided according to the levels of the most intricate to the most lowly. Generally element that lies at the highest level is called "points of focus" (SAATY, 2000).

Proximity to the truth of solutions with AHP method

Four conditions must be met that the solutions offered to solve the problem by AHP method be closer to the truth. These terms are reciprocity, homogenization, logical and continuity of the union (SAATY, 1994).

Reciprocity

As the need of the matrix twain comparison structure, comparison of elements w_i and w_j is done twice. At first it evaluates how many times the element w_i is important by element w_j then evaluate how much more the element w_i is important by element w_j. Because comparison of two elements made in the same period of time, naturally follows that if *a* component for example is twice more important than item *b* then it is indisputable that the item *b* is ½ times more important than the item's *a*.

$$a_{ij} = \frac{1}{a_{ji}} = \left(a_{ji}\right)^{-1}$$

Homogeneity condition

Homogeneous elements belonging to a particular class should be grouped together. So comparisons can be made between homogeneous elements and most importantly indicative table can be used to set numerical rating from 1 to 9. For example, it is illogical to compare the size between basketball ball and sun so that it is impossible to use indicative table of numbers used from 1 to 9. Since all the elements included in two comparisons, higher limit and lower limit (K) reads:

$$\frac{1}{K} \leq a_{ij} \leq K, \qquad K > 0 \ (i, j = 1..n)$$

In twain comparison matrix being that we will always assess digit largest and smallest for any matrix we have a constant (K).

Near consistency

As indicated above, the homogeneity of the elements to be included in the twain comparing influences the consistency of the matrix. On the other hand because the matrix is the result of a certain rating, it expresses the implemented opinion or the consistency of present residence. Calculating the consistency of evaluation of elements that are part of the established hierarchical structure shows how close to the truth is the evaluation. As expressed above conditions required that a matrix be consistent are shown below:

$$a_{ik} = a_{ij} \cdot a_{jk}, \qquad a_{ij} = \frac{1}{a_{ji}}$$

($W_i = 2W_j$) dhe ($W_j = 3W_k$) atëherë ($W_i = 6W_k$)

The possibility of being all in a matrix is very small. But when we make an assessment taking into account all these it may come to a close matrix that is fully consistent matrix. Variance of matrix should not exceed 10% (generally accepted 5% for n = 3 and 8 % for n = 4, for n ≥5 %10) (SAATY, 1994).

Uniform Continuity

Twain comparing matrix W_i (i = 1, 2 ... n), as a function of a_{ij} must be sensitive to small changes in a_{ij} so that the proportional value of W_i / W_j, produce good forecasts versus a_{ij}. Namely whether in the hierarchy formed in a matrix derived from no consistency and this condition is caused due to wrong assessment, to enable the consistency of the matrix the error in the assessment must be found and repair. These repairs are effective when W_i is sensitive to small changes in a_{ij}.

Functioning of Analytic Hierarchy Process

Analytical Hierarchy Process is a decision-making process. This process consists of three stages, during the formation of the hierarchy, the calculation of the final consistency and evaluation of results. These steps will be explained below in a row.

The formation of the hierarchy

Analytic hierarchy process mimics the analytical thinking way of the human. To get a healthy decision problems are evaluated by dividing into smaller parts. This fragmentation process continues until the causes of the problem is clarified. From here we understand that the complexity of the problem and the level of details affect in the classification and separation of levels of hierarchy for the decision to be taken (Zahedi F., 1986).

AHP method starts with selection of options and criteria that will make up the hierarchy of decision making about the problem we have in focus (Steiguer, 2003). Once the problem is identified, the desired decisions taken in connection with this problem are defined and these decisions are accepted as objectives. The target set at the highest peak of the hierarchy and then all of the elements belonging to the problem shared by level of importance and homogenization are conditioned by a level criteria specified above (Steiguer, 2003).

Formation of the twain comparison matrices

While using AHP method for the problems, to determine the approximate importance of the criteria and sub-criteria after the formation of hierarchical model, we must create twain comparison matrices (Sipahi S, 2002). The importance value of elements while twain comparing is defined according to the above level criteria.

Comparison matrix between elements is a square matrix of dimension n x n. The values at diagonal of the matrix take the value 1 by the matrix components. The comparison matrix is shown below.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Components on the diagonal of comparative matrix take the 1 (one) value because $i = j$. In this case the question item compared with itself. Comparison of elements become one by one taking in advance the level of importance of each of them.

All comparisons are made to the values remain above the diagonal that has the values 1 in the comparative matrix. To remain components which are below the diagonal that would normally be sufficient to use formula number 1.

$$a_{ji} = \frac{1}{a_{ij}} \qquad (1)$$

Determination of the priority values in comparison doubles

Twain comparing generally is a natural process of matching elements which are placed according to criteria based on people's preferences and can be explained by the sensitivity, order of importance or their consent (Saaty, 2001). Comparative Matrix shows elements within a certain logic by level of importance. So double values in the matrix comparisons show the gravity value for each element priority, using mathematical manipulations. But for all relevance within elements, namely to determine the distribution of importance in percentage, use the columns of vectors generated in the comparative matrix. So to determine all relevance criteria column vector formed with the number *n b* and *n* component (Yaralıoğlu, 2001). This vector is shown below:

$$B_i = \begin{bmatrix} b_{11} \\ b_{21} \\ . \\ . \\ . \\ b_{n1} \end{bmatrix}$$

In calculating the column vector *B* we can use formula number 2. That is the formula used in the twain comparison matrix, evaluation of every element in the same column is divided by total values which are in each column:

$$b_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \qquad (2)$$

For example, if comparative matrix *A*, which shows the comparison with each other elements of assessment, is defined as follows and are required to calculate the vector B_1.

$$A = \begin{bmatrix} 1 & 1/3 & 5 \\ 3 & 1 & 4 \\ 1/5 & 1/4 & 1 \end{bmatrix}$$

In this case the element b_{11} of vector B_1 will be calculated $b_{11} = \frac{1}{1+3+0,2}$

It is done at the same way for the other elements of vector B_1 so the vector obtained as follows. When assemble the components of the column vector we see that the total is 1 (one).

$$B_1 = \begin{bmatrix} 0,238 \\ 0,714 \\ 0,048 \end{bmatrix}$$

When you repeat the steps explained above in the values of other elements, we will have the so many *B* column vectors as the number of elements. When we collect according to the format of the matrix all the numbers *n* in the benefit column vector *B*, the shown below *C* matrix will form which is a normalized matrix.

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}$$

If we consider the above example matrix C will be as follows:

$$C = \begin{bmatrix} 0{,}238 & 0{,}210 & 0{,}500 \\ 0{,}714 & 0{,}632 & 0{,}400 \\ 0{,}048 & 0{,}158 & 0{,}100 \end{bmatrix}$$

Using normalized matrix *C* can obtain the value of the importance by percentages of types by elements. For this, as shown in the formula number 3, taken the arithmetic mean of the components of the lines formed in the normalized matrix *C* and derived vector from this column vector of *W* is called priority vector.

$$w_i = \frac{\sum_{j=1}^{n} c_{ij}}{n} \qquad (3)$$

Vector *W* is as shown below:

$$W = \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{bmatrix}$$

When choosing the example above priority elements of the vector can be calculated as follows. In this case the value of three factors together they will have approximately these values. The first factor 32%, 58% the second factor and the third factor 10%.

$$W = \begin{bmatrix} \frac{0{,}238 + 0{,}210 + 0{,}500}{3} \\ \frac{0{,}714 + 0{,}632 + 0{,}400}{3} \\ \frac{0{,}048 + 0{,}1580 + 0{,}100}{3} \end{bmatrix} \cong \begin{bmatrix} 0{,}32 \\ 0{,}58 \\ 0{,}10 \end{bmatrix}$$

Calculation of the consistency

Although AHP method is a consistent system itself, the authenticity of the results will be dependent on compliance with comparisons between the elements that makes the decision maker. If the decision maker is shown contradictory assessments, he could not find where the dot is bigger discrepancy when he comes back. While AHP method using the advantage of double aligning assessments not only find the discrepancy but also shows that which may be appropriate values (Saaty, 1990). AHP method proposes a process to measure compliance of these comparisons. In the end it gives us the opportunity to test the consistency of the priority vector that is the degree of consistency (CR-consistency Rate) then the comparison made between the elements one by one. Core calculation of CR based on AHP method to compare the number of elements with a coefficient (λ) called fundamental value. To calculate the basic value of coefficient (λ) at the beginning it have to obtain column vector *D* which is obtained by multiplying the priority vector *W* with comparison matrix *A*.

$$D = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} x \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{bmatrix}$$

As defined in the formula number 4, acquired basic value (*E*) for each element in the evaluation of reciprocal elements between column vector *W* and *D*. The formula number 5, which include the arithmetic average of these values gives value basic (λ) in connection with the comparison. $E_i = \frac{d_i}{w_i}$ ($i = 1,2,..., n$) (4), $\lambda = \frac{\sum_{i=1}^{n} E_i}{n}$ (5)

After the calculation of basic values (λ) with the help of formula number 6 we can find Consistency Index (CI-consistency Index).

$$CI = \frac{\lambda - n}{n - 1} \quad (6)$$

While in the last step of CI is obtained CR by dividing the standard adjustment value shown in Table 1 called random indicator (Random Index RI). In Table 1 is selected the value which correspond to the number of elements. For example, the value of RI to be used in a comparison with the 3 elements under table 3 will be 0:58.

Table 1: *Value of Random Index (RI)*

n	1	2	3	4	5	6	7	8	9	10	11	12	13
Treguesi i rastësishëm	0	0	0,58	0,9	1,12	1,24	1,32	1,41	1,45	1,49	1,51	1,48	1,56

Source: Oğuzlar, 2007.

$$CR = \frac{CI}{RI} \quad (VII)$$

In cases where the estimated value of CR is less than 0.10 then is clear that the comparisons made by the decision maker are consistent. If the value of CR is greater than 0.10 then we have an error in calculation method AHP or instability in the comparisons made by the decision maker.

Finding the importance of the distribution from percentages degree for each element

At this stage determined the distribution of importance from the degree of percentage rate for each element. Saying in other words, matrix procedures and one by one comparison will be repeated as many time as *n* number of elements. This time the dimensions of comparative matrix *G* to be used in the decision points for each element will be *m x m*. After each comparison procedure column vector S is obtained that shows the distribution of the importance degree and by percentage degree and decision points of the evaluated item by the dimension *m x l*. This column vector is shown below:

$$S_i = \begin{bmatrix} s_{11} \\ s_{21} \\ . \\ . \\ . \\ s_{m1} \end{bmatrix}$$

Distribution of the end points of the decision.

At this stage of the decision matrix *K* with dimension *m x n* formed by the columns of the vector *S* with *n* pieces with dimension *m x l* explained above. The decision matrix shown as follows:

$$K = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1n} \\ s_{21} & s_{22} & \dots & s_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ s_{m1} & s_{m2} & \dots & s_{mn} \end{bmatrix}$$

In conclusion, when the decision matrix *W* is multiplied following the column vector (vector of priority) *S* we obtain a column vector *L* with *m* elements. Column vector *L* gives the percentage distribution of decision points. In other words the total value of the elements of the vector is 1. This distribution also provides the order of importance of the decision points.

$$L = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1n} \\ s_{21} & s_{22} & \dots & s_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ s_{m1} & s_{m2} & \dots & s_{mn} \end{bmatrix} x \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{bmatrix} = \begin{bmatrix} l_{11} \\ l_{21} \\ . \\ . \\ . \\ l_{m1} \end{bmatrix}$$

Method of Analytic Hierarchy Process is developed by passing through all the stages described above.

Conclusions

Pattern formed with the help of AHP method being simpler than the older models in use, a model of renewable over time and open to changes is more likely to be used in a lot social science fields in the future. All elements found in the hierarchical structure of the model form with the help of AHP method, after passing in the process of twain comparing, prioritize each criterion. Given that the criteria used in the model show changes from person to person for every person have different point advantage. Thanks such a model it's possible to create accurate decision making. Consequently, this model is quite convenient for the banking system, which needs accurate decision (granting loans, credit cards, etc.).

Bibliography:

[1] De Steiguer J. E., Jennifer Duberstein, Vincent Lopes, "The Analytic Hierarchy Process, as a Means of Integrated Watershed Mnagement" **Proceedings of the First Interagency Conference on Research in the Watersheds,** USA, 2003, 736–740, p.738.

[2] Saaty Thomas L., "How to Make Decision: The Analytic Hierarchy Process**", European Journal of Operational Research**, 48, North-Holland, 1990, p.9–26.

[3] Saaty Thomas, "Deriving the AHP 1-9 Scale from First Principles", **ISAHP Berne, Switzerland 2-4,** August 2001, p.1.

[4] Saaty, **Fundamentals of Decision Making and Priority,** p.58.

[5] Sipahi S. ve Berber A, "Dönüşümsel liderlik Perspektifinin Analitik Hiyerarşi prosesi tekniği ile Analizi", **İ.Ü. İşletme Fakültesi Dergisi**, C:31, numri 1, Prill 2002

[6] Yaralıoğlu Kaan, "Performans Değerlendirmede Analitik Hiyerarşi Proses", **Dokuz Eylül Üniversitesi, İİBF Dergisi**, Cilt: 16, numër .I., p.131.

[7] Yaralıoğlu Kaan, "Performans Değerlendirmede Analitik Hiyerarşi Proses", **Dokuz Eylül Üniversitesi İ.İ.B.F. Dergisi**, Cilt:16, Sayı:1, İzmir, 2001, p.137–145.

[8] Zahedi F. "The Analytical Hierarchy Process: A Survey of the Method and its Applications", **Interfaces**. 1986, 16 (4), 96–108.