



Progressive Income Taxation and Economic Growth with Endogenous Labor Supply and Public Good

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Abstract

This study constructs a sophisticated economic growth model centered on the gender division of labor and endogenous labor supply, specifically within the structural framework of a nonlinear progressive income taxation system. In this model, tax revenues are strategically allocated to the provision of public goods, creating a dual-structure economy composed of a primary private production sector and a taxpayer-supported public sector. By integrating these complex macroeconomic elements, the research effectively describes the intricate dynamic interactions between long-term economic growth, gender-based labor distribution, and progressive fiscal policies. To validate the theoretical framework, the study utilizes advanced numerical simulations to demonstrate the existence of a stable economic equilibrium and to map the overall trajectory of the dynamic system. Furthermore, a comprehensive sensitivity analysis is conducted to examine how fluctuations in various policy and behavioral parameters influence the long-term motion, stability, and transition paths of the broader economic system. Ultimately, this work provides critical insights into how tax structures, public goods financing, and gendered labor dynamics coalesce to shape the economic evolution and structural equilibrium of modern societies, offering macroeconomists and fiscal policymakers a robust analytical framework for evaluating the long-term socio-economic impacts of progressive tax reforms.

Keywords: progressive income taxation; economic growth; endogenous labor supply; public good

1. Introduction

To understand modern economies it is important to study dynamic interdependence between economic growth and public investment (Barro, 1990; and Turnovsky, 2000,

2004). Public policies have important implications for private consumption, saving, capital formation and production. The purpose of this study is to study the dynamic relationship between government's spending and private consumption within a dynamic general equilibrium framework. We study a neoclassical growth model "with two prevalent features observed in developed economies: progressive income taxation together with utility-generating public spending", Chen and Guo (2014: 174). Nevertheless, different from the traditional models with the Ramsey utility (e.g., Barro and Sala-i-Martin, 1992; Futagami *et al.*, 1993; Glomm and Ravikumar, 1994, 1997; Agénor, 2011; Baier and Glomm, 2001; Palivos *et al.*, 2003; Greiner, 2007; Hu *et al.*, 2008; Kamiguchi and Tamai, 2011; and Chen and Guo, 2013, 2014), this study applies Zhang's utility function (Zhang, 1993). We show that the dynamic model with Zhang's approach has a stable equilibrium point, implying that one can effectively conduct comparative dynamic analysis.

We examine a dynamic interdependence between labor supply and economic growth with nonlinear progressive income taxation. The tax income is spent on supplying public goods. We introduce gender and public goods into the neoclassical growth theory. Although gender differences have long been taken into account in economic analysis at the microeconomic level, it is only recently that gender issues have been examined in macroeconomic analytical frameworks. Stotsky (2006) identifies a number of phenomena related to gender differences and economic behavior: (1) gender-based differences can influence macroeconomic variables, such as aggregate consumption, savings; (2) these differences may also affect the behavior of governments; (3) women tend to devote a larger share of household resources to the households' basic needs and the children's fostering; (4) women tend to have a higher propensity to save and to invest in productive activities and show greater caution in saving and investing; (5) women's lack of education and other economic and social opportunities, both absolutely and relative to men, inhibits economic growth. Although there have been a number of attempts to modify neoclassical consumer theory to deal with economic issues about endogenous labor supply, family structure, working hours and the valuation of traveling time with endogenous sexual division of labor and consumption (Becker, 1976; Gomme *et al.*, 2001; Campbell and Ludvigson, 2001; Gutierrez, 2003; Tassel, 2004; Stotsky, 2006), economics still needs proper analytical frameworks to deal with gender division of labor and economic growth theory with capital accumulation, public goods and different fiscal policies. As gender differences are interactive with economic growth and government fiscal policies affect economic growth and households' behavior, it is reasonable to assume that gender differences in behavior that result from the private decision or are affected by public policies should lead to different outcomes in economic growth. But Stotsky (2006) observes that there are few formal models which take account of impact of fiscal policies on gender differences. This study attempts to make a contribution by proposing a formal growth model of describing dynamic effects of progressive income taxation on economic growth and gender division of labor. In

examining behavior of the model, our attention is focused on the numerical simulations of a calibrated economy. Simulation enables us to see movements of different variables during lengthy transition periods. We highlight the dynamic effects of fiscal policy and the tradeoffs these involve for economic performance.

This study is conducted in an analytical framework for small open economies. There is a large number of the literature on economics of open economies (e.g., Obstfeld and Rogoff, 1996; Lane, 2001; Kollmann, 2001, 2002; Benigno and Benigno, 2003; Galí and Monacelli, 2005; Uya, *et al.* 2013; and Ilzetzki, *et al.* 2013). We follow this tradition in dealing with dynamic interdependence between economic growth, public goods, and progressive income taxation. Rather than following the main frameworks in modeling household behavior in economic growth theory with wealth accumulation, we use Zhang's utility function to deal with behavior of the households. It is well known that the Solow model is the starting point for almost all analyses of economic growth (Solow, 1956). The Solow model does not provide a mechanism of endogenous savings. Another important approach to the household behavior is the representative agent growth model with Ramsey's utility function (Ramsey, 1928; Cass, 1965; Koopmans, 1965). One of the problems of this approach is that it makes the analysis intractable even for a simple economic growth problem. Another approach in economic modeling is the so-called OLG approach (Diamond, 1965, Samuelson, 1959). The approach is a discrete version of the continuous Ramsey approach (Azaridias, 1993). This study will model behavior of households with an alternative approach proposed by Zhang in the early 1990s (Zhang, 1993). This study is an extension of the growth models with public goods proposed by Zhang (2010, 2014). In Zhang's models, all the tax rates are fixed. This study makes taxation on the household's income an endogenous variable. This paper is organized as follows. Section 2 introduces the basic model with wealth accumulation, gender division of labor, externalities, public goods, and congestion. Section 3 examines dynamic properties of the model. Section 4 simulates the motion of the economic system and demonstrates effects of changes in some parameters on the economic system. Section 5 concludes the study.

2. The basic model

This section develops a small-open three-sector growth model with endogenous wealth and public goods. We consider that the open economy can import goods and borrow resources from the rest of the world or exports goods and lend resources abroad. There is a single good, called industrial good, in the world economy and the price of the industrial good is unity. The rate of interest, r^* , is fixed in international market. Capital depreciates at a constant exponential rate, δ_k . The economy has one production sector and one public goods sector (Zhang, 2010). Most aspects of the production sector are similar to the standard one-sector growth model (Burmeister and Dobell, 1970; Barro and Sala-i-Martin, 1995). The population is constant and homogenous; each worker is employed in either of the two sectors. Households own assets of the economy and

distribute their incomes to consume and save. Production sectors or firms use labor and capital inputs to produce goods. Exchanges take place in perfectly competitive markets. Factor markets work well and factors are fully utilized at every moment. Saving is undertaken only by households. Labor force and capital stock are distributed between the two sectors. Household activities are conducted within houses as well as outside doors. For simplicity, we assume that the country has N_0 identical families. Each family consists of four members - father, mother, son and daughter. The total population of the country is equal to $4N_0$. It can be seen that on the basis of the literature of growth models with endogenous fertility (for instance, Barro and Becker, 1989; Raut, 1992; Conde-Ruiz et al., 2010; Zhang, 2015), it is possible to generalize our model to take account of endogenous fertility. It is assumed that only the adults work. The children get educated before they get married and join the labor market. We assume that the husband and wife pass away at the same time. When the parents pass away, the son and the daughter respectively find their marriage partner and get married. The properties left by the parents are shared equally among the male and female children. The children are educated so that they have the same human capital as their parents. When a new family is formed, the young couple joins the labor market and has two children. As all the families are identical, the family structure is invariant over time under these assumptions. Let subscripts $j = 1$ and $j = 2$ stand for man and woman respectively. Let $T_j(t)$ stand for the work time of a representative household of gender j and $N(t)$ for the flow of labor services used at time t for production. We have $N(t)$ as follows

$$N(t) = \sum_{j=1}^2 h_j N_0 T_j(t),$$

where h_j is the level of human capital of gender j . We assume h_j to be fixed. We also have $N(t) = N_0 \tilde{T}(t)$, where $\tilde{T}(t) \equiv \sum_{j=1}^2 h_j T_j(t)$.

The production sector

We specify the production function of the production sector as follows

$$F(t) = AK_i^{\alpha_i}(t)N_i^{\beta_i}(t), \quad \alpha_i + \beta_i = 1, \quad \alpha_i, \beta_i > 0,$$

where α_i and β_i are parameters. Like Chen and Guo (2014), we omit possible effects of public goods on productivity. The marginal conditions

$$r^* = \frac{\alpha_i F(t)}{K_i(t)} - \delta_k, \quad w(t) = \frac{\beta_i F(t)}{N_i(t)}, \quad w_j(t) = h_j w(t), \quad (1)$$

where r^* is the rate of interest fixed in the international market, $w_j(t)$ is the wage rate per unit of time for gender j . From (1), we have $w_1(t)/w_2(t) = h_1/h_2$. This relation is a consequence of assumption that there is no gender discrimination in the sense that any one is paid per unit of time according to one's work efficiency. In reality, this is a strict requirement as there is gender discrimination in labor markets (Beneria, 1995, 1999; Banerjee, 2008; Booth, 2009; Dong and Zhang, 2009).

The disposable income and budget constraint

Let $\bar{k}(t)$ stand for the capital stock owned by a household. To explain our approach to consumer behaviour, we notice that when there is no taxation on the household's current income the household income from the interest payment and the wage payments is

$$y_0(t) = r^* \bar{k}(t) + w_1(t)T_1(t) + w_2(t)T_2(t). \quad (2)$$

In the studies by Guo and Lansing (1998) and Chen and Guo (2014), the income tax rate $\tau(t)$ is taken on the following form

$$\tau(t) = 1 - \tau_0 \left(\frac{y^*}{y_0(t)} \right)^a,$$

where y^* is the steady-state level of per capita income, which is taken as given by each household. In our study we don't require a given y^* . We introduce the progressive tax rate $\tau_h(t)$ as a function of $y_0(t)$ as follows

$$\tau_h(t) = \tau_0 + \tau_1 y_0^a(t), \quad 1 > \tau_0, a > 0, \tau_1 > 0. \quad (3)$$

In the case of $a > 0$ the tax rate is increased as the per capita income is increased. The tax schedule is said to be progressive. In the case of $a = 0$ the tax schedule is called flat. Many studies assume a constant tax rate of income or a flat consumption tax (e.g., Cazzavillan, 1996; Zhang, 2000; Raurich, 2003; Fernández *et al.* 2004; Chen, 2006; Guo and Harrison, 2008). We also mention that Lloyd-Braga *et al.* (2008) introduce progressive consumption taxation. A more general approach should take account all possible types of taxation. For simplicity of analysis, this study is focused on progressive

income taxation. The representative household's current income $y(t)$ with the given tax rate is

$$y(t) = \bar{\tau}_h(t) y_0(t), \quad (4)$$

where $\bar{\tau}_h(t) = 1 - \tau_h(t)$. We neglect the possibility that man and woman may be subject to different tax policies. The total value of wealth that a representative household can sell to purchase goods and to save is $\bar{k}(t)$. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per household disposable income of the household is defined as the sum of the current income and the wealth available for purchasing consumption goods and saving

$$\hat{y}(t) = y(t) + \bar{k}(t).$$

The disposable income is used for saving and consumption. At each point in time, the household would distribute the total available budget between savings $s(t)$ and consumption of goods $c(t)$. The budget constraint is given by

$$c(t) + s(t) = \hat{y}(t).$$

This equation means that consumption and savings exhaust the consumers' disposable income. Denote $\bar{T}_j(t)$ the leisure time at time t and the (fixed) available time for work and leisure by T_0 . The time constraint is expressed by

$$T_j(t) + \bar{T}_j(t) = T_0, \quad j = 1, 2.$$

Substituting this time constraint into the disposable income yields

$$\hat{y}(t) = \bar{y}(t) - \bar{\tau}_h(t) w_1(t) \bar{T}_1(t) - \bar{\tau}_h(t) w_2(t) \bar{T}_2(t), \quad (5)$$

where

$$\bar{y}(t) \equiv \bar{k}(t) + \bar{\tau}_h(t) r^* \bar{k}(t) + \bar{\tau}_h(t) w_1(t) T_0 + \bar{\tau}_h(t) w_2(t) T_0.$$

Substituting (5) into the budget constraint yields

$$\bar{\tau}_h(t) w_1(t) \bar{T}_1(t) + \bar{\tau}_h(t) w_2(t) \bar{T}_2(t) + c(t) + s(t) = \bar{y}(t). \quad (6)$$

The utility function

At each point in time, the representative household decides the three variables subject to the budget constraint. We assume that utility level $U(t)$ is dependent on the leisure times, the consumption level of commodity, and savings as follows

$$U(t) = u(G(t), t) \bar{T}_1^{\sigma_{01}}(t) \bar{T}_2^{\sigma_{02}}(t) c^{\xi_0}(t) s^{\lambda_0}(t), \quad \sigma_{01}, \sigma_{02}, \xi_0, \lambda_0 > 0,$$

where u is a time-dependent variable, σ_{01} , σ_{02} , ξ_0 and λ_0 are called respectively the husband's and wife's propensities to stay at home, the family's propensities to consume good and to hold wealth. For simplicity, we specify the utility function with the Cobb-Douglas form. A detailed explanation of the approach and its applications to different dynamic problems are provided in Zhang (2009).

The household's optimal behaviour

Maximizing U_j subject to budget constraint (5) yields

$$\begin{aligned} \bar{\tau}_h(t) w_1(t) \bar{T}_1(t) &= \sigma_1 \bar{y}(t), \quad \bar{\tau}_h(t) w_2(t) \bar{T}_2(t) = \sigma_2 \bar{y}(t), \quad c(t) = \xi \bar{y}(t), \\ s(t) &= \lambda \bar{y}(t), \end{aligned} \quad (7)$$

where

$$\rho \equiv \frac{1}{\sigma_{01} + \sigma_{02} + \xi_0 + \lambda_0}, \quad \sigma_1 \equiv \rho \sigma_{01}, \quad \sigma_2 \equiv \rho \sigma_{02}, \quad \xi \equiv \rho \xi_0, \quad \lambda \equiv \rho \lambda_0.$$

From (4), we have

$$\frac{\bar{T}_1(t)}{\bar{T}_2(t)} = \frac{\sigma_1 w_2(t)}{\sigma_2 w_1(t)}.$$

We see that the ratio of time at home between man and woman is positively related to the ratio of man's and woman's propensity to stay at home and negatively related to the ratio of man's and woman's wage rates. Some studies examine interdependence between growth and time allocation in the literature of economic development, predicting reallocation of labor from households to the market in association of economic growth (for instance, Becker, 1965; Goodfriend and McDermott, 1995; Kelly, 1997; Edmonds and Pavcnik, 2006; and Ferber and Green, 2007). There is an immense body of empirical and theoretical literature on economic growth with time distribution between home and non-home economic and leisure activities (e.g., Greenwood and Hercowitz, 1991; Benhabib and Perli, 1994; Ladrón-de-Guevara *et al.* 1997; Rupert *et al.* 2001; Cambell and Ludvigson, 2001; Vendrik, 2003; and Chesters *et al.* 2009). This study shows how the progressive taxation policy affects the gender division of labor and leisure.

The wealth accumulation

According to the definitions of $s(t)$ and $\bar{k}(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t). \quad (8)$$

This equation simply means that change in the wealth is equal to saving minus dissaving.

The public sector

We now describe the public sector. In this model, we assume that the public sector is financially supported by the government's tax income. The capital stocks and workers employed by the public sector are paid at the same rates that the private sector pays the services of these factors. The public sector has income as follows

$$I_p(t) = [r^* \bar{k}(t) + w_1(t)T_1(t) + w_2(t)T_2(t)]\tau_h(t)N_0. \quad (9)$$

We assume that the public sector supplies public goods by utilizing capital, $K_p(t)$, and labor force, $N_p(t)$, as follows

$$G(t) = A_p K_p^{\alpha_p}(t) N_p^{\beta_p}(t), \quad A_p, \alpha_p, \beta_p > 0, \quad \alpha_p + \beta_p = 1.$$

For the given tax rates, the public sector is faced with the budget constraint

$$w(t)N_p(t) + (r^* + \delta_k)K_p(t) = I_p(t). \quad (10)$$

The government may have various objectives in providing public services. In this study it is assumed that public sector behaves effectively in the sense that it will use the available resource to maximize public services. Maximizing public services under the budget constraint yields

$$\frac{\alpha_p N_p(t)}{\beta_p K_p(t)} = \frac{r^* + \delta_k}{w}. \quad (11)$$

The factors are fully employed

$$K_i(t) + K_p(t) = K(t), \quad N_i(t) + N_p(t) = N(t). \quad (12)$$

We thus built the model with progressive income taxation. In the rest of the paper, we will examine properties of the model and see how changes in different parameters will affect the economic system.

3. The behavior of the model

This section studies dynamics of the model. First, we show that the motion of the entire economic system can be described by a differential equation with the tax rate as the variable. The following lemma is checked in the Appendix.

Lemma

The dynamics of the economic system is governed by the following one differential equation with the tax rate as the variable

$$\dot{\tau}_h(t) = \bar{\Lambda}(\tau_h(t)), \quad (13)$$

where $\bar{\Lambda}(t)$ is a function of $\tau_h(t)$ defined in the appendix. The values of all the other variables are uniquely determined as functions of $\tau_h(t)$ at any point in time by the following procedure: $N_p(t)$ by (A5) $\rightarrow K_p(t)$ by (A1) $\rightarrow I_p(t) = wN_p(t)/\beta_p \rightarrow W$ by (A8) $\rightarrow \bar{k}(t)$ by (A8) $\rightarrow \bar{K}(t) = \bar{k}(t)N_0 \rightarrow N(t)$ by (A6) $\rightarrow K_i(t)$ by (A9) $\rightarrow N_i(t)$ by (A9) $\rightarrow y_0(t)$ by (A7) $\rightarrow T_j(t)$ by (A5) $\rightarrow \bar{T}_j(t)$ by (A5) $\rightarrow \bar{y}(t)$ by (A4) $\rightarrow F(t)$ by (A2) $\rightarrow K(t)$ by (A4) $\rightarrow G(t)$ by the definition $\rightarrow c(t)$ and $s(t)$ by (A7) $\rightarrow w_j$ by (1).

As the expressions are tedious, it is difficult to interpret the analytical results. For illustration, we simulate the model to demonstrate dynamic properties of the model. We specify the parameter values as follows

$$\begin{aligned} r^* &= 0.05, \quad \alpha_i = 0.32, \quad \alpha_p = 0.4, \quad A = 1.5, \quad A_p = 0.9, \quad N_0 = 100, \\ h_1 &= 3, \quad h_2 = 2.6, \quad T_0 = 12, \quad \xi_0 = 0.15, \quad \lambda_0 = 0.8, \quad \sigma_{01} = 0.1, \\ \sigma_{02} &= 0.15, \quad \tau_0 = 0.03, \quad \tau_1 = 0.2, \quad a = 0.2, \quad \delta_k = 0.08. \end{aligned} \quad (14)$$

The rate of interest is 0.05. Man's propensity to stay at home is lower than woman's propensity to stay at home. Man's human capital is higher than woman. The total productivities of the two sectors are specified at 1.5 and 0.9, respectively. Although the specified values are not based on empirical observations, the choice does not seem to be unrealistic. For instance, some empirical studies on the US economy demonstrate that the value of the parameter, α , in the Cobb-Douglas production is approximately equal to 0.3 (for instance, Miles and Scott, 2005, Abel et al, 2007). With regard to the technological parameters, what are important in our study are their relative values. This is similarly true for the specified preference parameters. With the initial conditions, $\tau_h(0) = 0.24$, the changes of the variables over time are plotted in Figure 1. As shown in the appendix, the wage rates of the man and woman are determined as functions of the rate of interest. The tax rate rises over time, which implies that the current income before tax is rising. Both man and woman reduce their working hours. The consumption level and wealth rise. The production levels of the two sectors are reduced over time. The trade balance is improved. The nation uses less capital and holds more wealth,

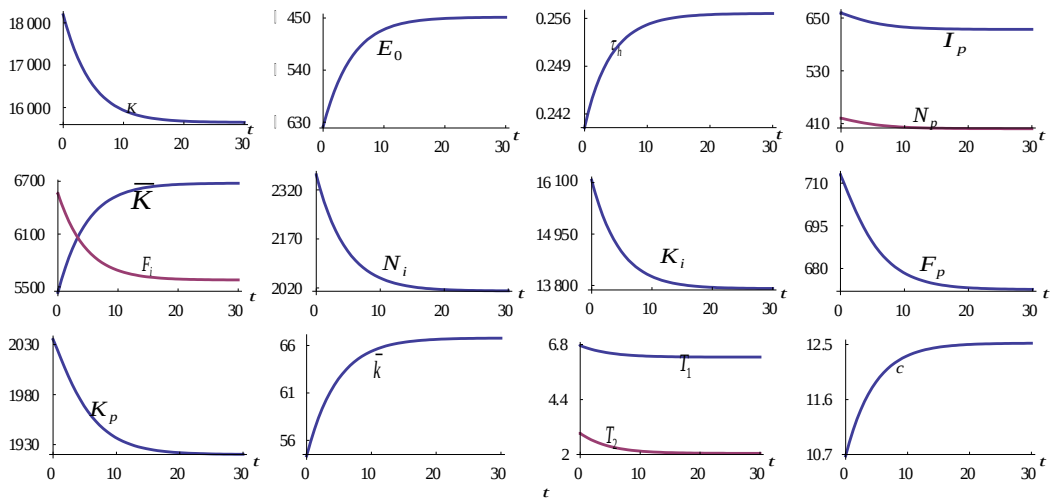


Figure 1. The Motion of the Dynamic System

The equilibrium values of the variables are given as follows

$$\begin{aligned} w_1 &= 5.66, \quad w_2 = 4.9, \quad N = 2409.3, \quad K = 15649.7, \quad \bar{K} = 6679.3, \\ E_0 &= -448.5, \quad \tau_h = 0.257, \quad I_p = 624, \quad F_i = 5577.7, \quad F_p = 672.6, \\ N_i &= 2010.9, \quad N_p = 398.4, \quad K_i = 13729.6, \quad K_p = 1920.1, \quad \bar{k} = 66.8, \\ T_1 &= 6.25, \quad T_2 = 2.05, \quad c = 12.5. \end{aligned}$$

The eigenvalue is -0.224 . The equilibrium point is stable. This result is important as it allows us to effectively conduct comparative dynamic analysis.

4. Comparative dynamic analysis

We now examine impact of changes in some parameters on the national economy. As we explicitly provided the procedure to simulate the motion, it is straightforward to make comparative dynamic analysis.

A fall in the rate of interest in the global market

First, we examine the case that the rate of interest is reduced as follows: $r^* : 0.05 \Rightarrow 0.04$. The simulation results are plotted in Figure 2. In the plots, a symbol $\bar{\Delta}$ stands for the change rate of the variable in percentage due to changes in the parameter value. As the capital cost falls in the global market, the wage rates are increased as follows

$$\bar{\Delta} w_1 = \bar{\Delta} w_2 = 3.84.$$

Both the husband and wife work less hours initially and work more hours in the long term. The tax rate and tax income fall. The household accumulates more wealth and consume more initially, but accumulates less wealth and consume less in the long term. The national economy employs less foreign capital initially but more in the long term.

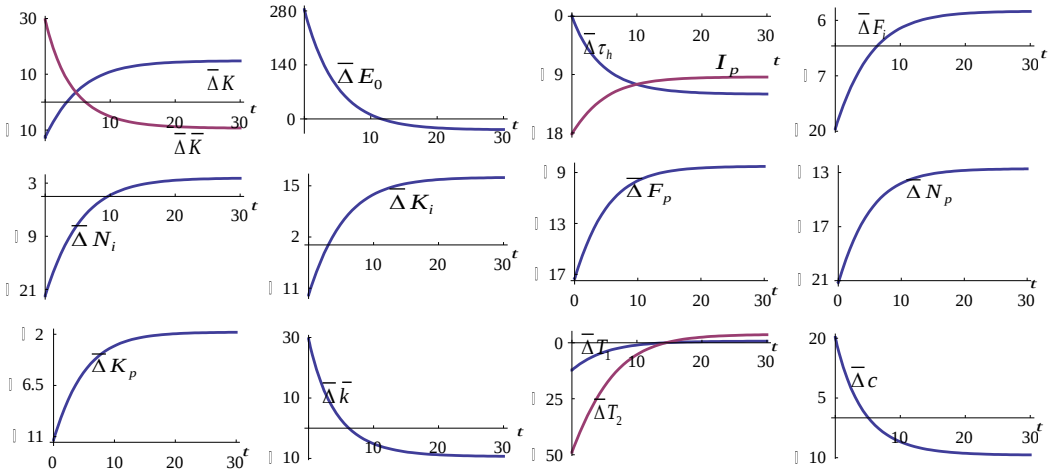


Figure 2. A Fall in the Rate of Interest

A rise in woman's human capital

We now increase woman's human capital as follows: $h_2 : 2.6 \Rightarrow 2.8$. The simulation results are plotted in Figure 3. Man's wage rate is not affected and the woman's wage rate is increased as follows

$$\bar{\Delta} w_1 = 0, \quad \bar{\Delta} w_2 = 7.69.$$

The wife works more hours and the husband works less. The tax rate is increased and the government gets more income. The two sectors' output levels and input factors are increased. The nation employs more capital and the nation has more wealth. The trade balance is improved.

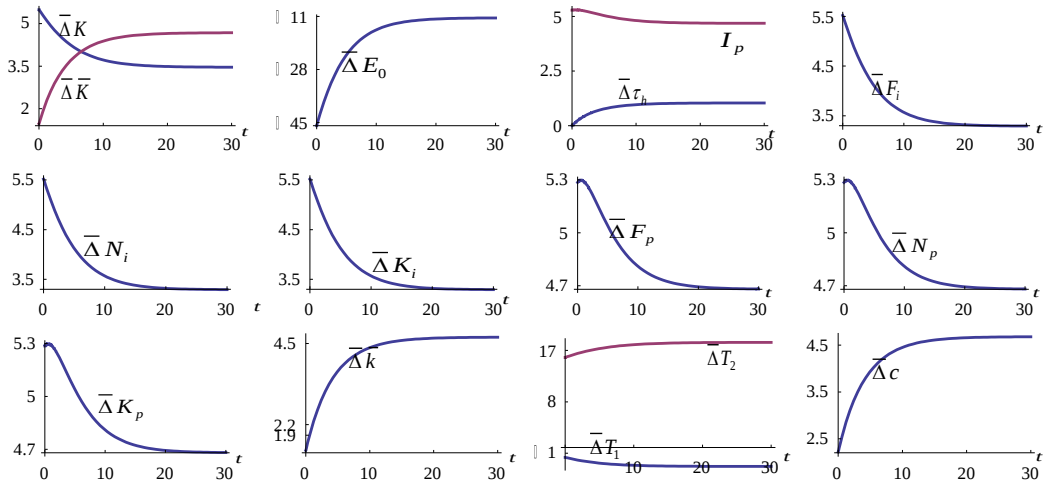


Figure 3. A Rise in Woman's Human Capital

The man's propensity to stay at home being enhanced

We now increase woman's human capital as follows: $\sigma_{01}: 0.1 \Rightarrow 0.12$. The simulation results are plotted in Figure 4. The wage rates are not affected. As the man tends to have stronger preference for staying at home, the man's working hours is reduced and the woman's working hours is augmented. The tax rate and tax income fall. The two sectors' output levels and input factors are lowered. The nation employs less capital and the nation has less wealth. The trade balance is improved. The household consumes less and has less wealth. Accordingly, the tendency for the man to stay longer hours without change in other variables such as human capital will worsen living conditions and lower national economic performance. It should be noted that many gender studies on development and gender division of labor tend to be focused on the strong increase in married women's labour force participation. Except well-noted reasons such as financial factors as well as demographic and socio-economic variables, another important factor is due to the change in household norm (which traditionally means that for a married couple living together according to the traditional life style, the husband earns the family income in the formal market, whereas the wife take care of household and the children, and hence is not involved in paid work). Vendrik (2003) shows that the traditional household norm seems to have dominated in many OECD countries up to the 1960s, but have been changed rapidly since then.

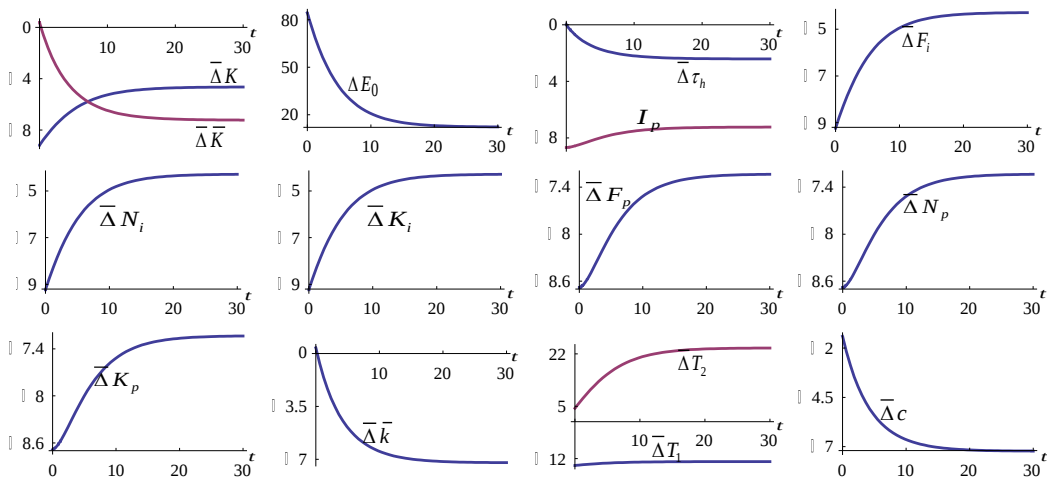


Figure 4. The Man's Propensity to Say at Home Being Enhanced

The income taxation being more progressive

We now consider the case that the taxation is more progressive as follows: $\alpha : 0.2 \Rightarrow 0.25$. The simulation results are plotted in Figure 5. The wage rates are not affected. The tax rate and tax income are increased. The public sector's output level and two inputs are augmented. Both the consumption level and wealth are lowered initially and enhanced in the long term. The man works longer hours initially and works almost the same hours in the long term. The woman works longer hours initially and works less hours in the long term. The trade balance is initially deteriorated and improved in the long term. The public sector's output level and two inputs are augmented initially and are reduced in the long term. The nation employs more capital initially and less in the long term.

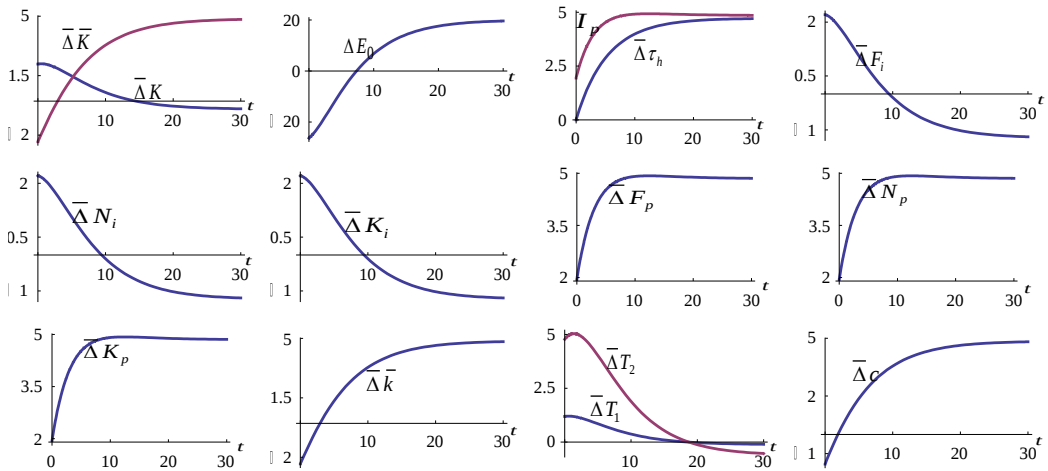


Figure 5. The Income Taxation Being More Progressive

The tax rate function's constant part being increased

We now increase the tax rate function's constant part as follows: $\tau_0 : 0.03 \Rightarrow 0.04$.

The simulation results are plotted in Figure 6. Comparing Figures 5 and 6, we see that the effects of making the taxation more progressive are qualitatively similar to the effects in this case.

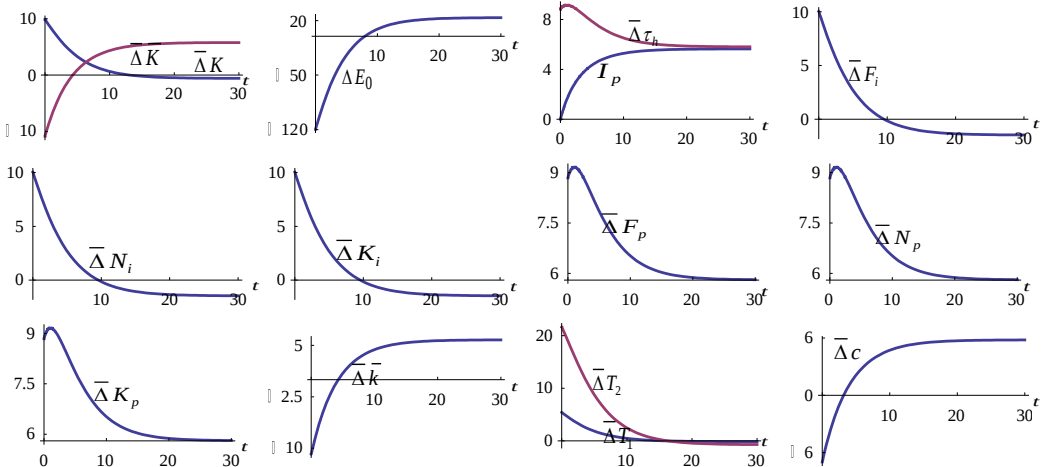


Figure 6. The Tax Rate Function's Constant Part Being Increased

A rise in the propensity to save

We now consider the case that the household increases the propensity to save as follows: $\lambda_0 : 0.8 \Rightarrow 0.81$. The simulation results are plotted in Figure 7. The wage rates

are not affected. The tax rate and tax income are increased. The public sector's output level and two inputs are augmented. The wealth is increased. The consumption level is lowered initially and enhanced in the long term. The man works longer hours initially and works almost the same hours in the long term. The woman works longer hours initially and works less hours in the long term. The trade balance is initially deteriorated and improved in the long term. The industrial sector's output level and two inputs are augmented initially and are reduced in the long term. The nation employs more capital initially and less in the long term.

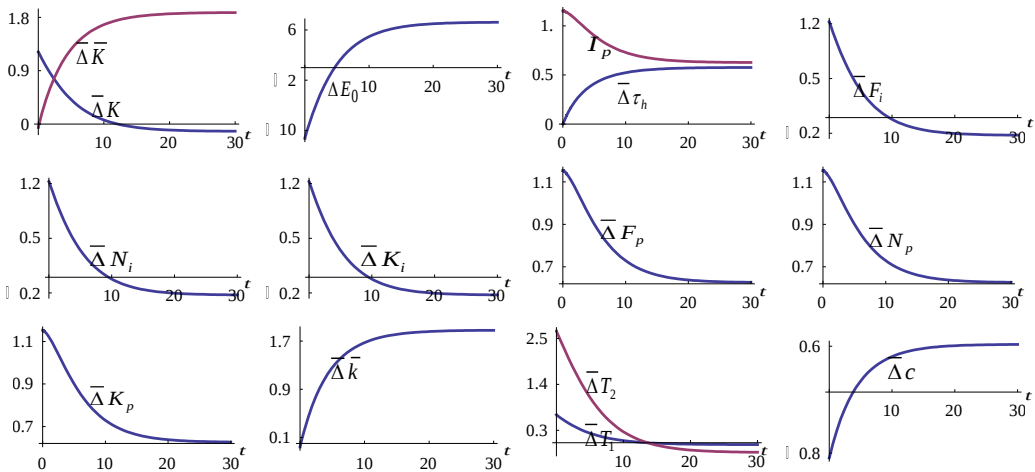


Figure 7. A Rise in the Propensity to Save

The total factor productivity being enhanced

We now consider the case that the total factor productivity is enhanced as follows: $A: 1.5 \Rightarrow 1.6$. The simulation results are plotted in Figure 8. The wage rates are increased. The tax rate and tax income are increased. The public sector's output level and two inputs are augmented. Both the consumption level and wealth are enhanced. Both man and woman work longer hours initially and work almost the same hours in the long term. The trade balance is deteriorated. The industrial sector's output level and capital input are augmented. The nation employs more capital.

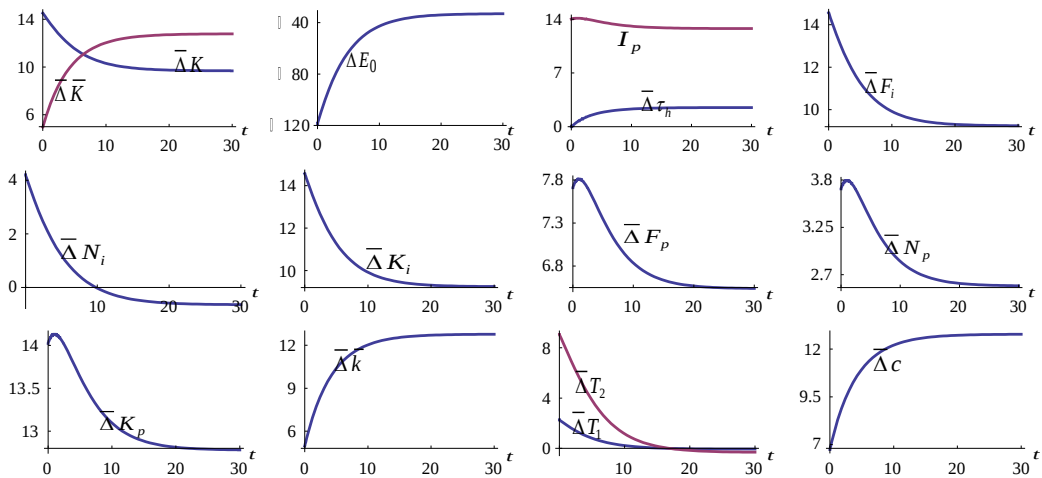


Figure 8. The Total Factor Productivity Being Enhanced

The total population being increased

The standard one-sector neoclassical growth model predicts that change in the population has no impact on per capita consumption and wealth. As the taxation is progressive in our model, our model predicts different reaction to change in the population. We now allow the population to be increased as follows: $N : 100 \Rightarrow 105$. The simulation results are plotted in Figure 9. The wage rates are not affected. The work hours are increased initially and are affected slightly in the long term. The tax rate and tax income are increased. The public sector's output level and two inputs are augmented. Both the consumption level and wealth are enhanced. The trade balance is improved. The industrial sector's output level and capital input are augmented. The nation employs more capital.

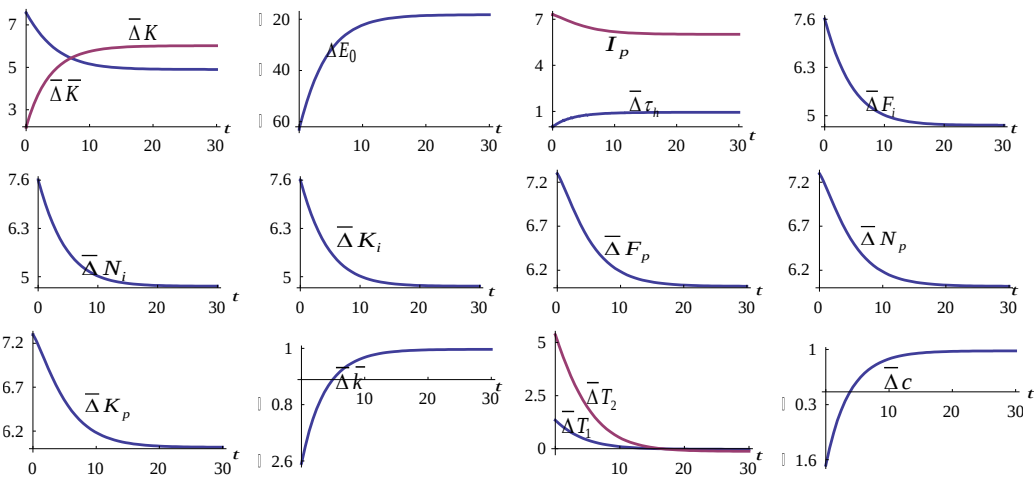


Figure 9. A Rice in the Population

5. Concluding remarks

This study developed a dynamic economic growth model of public goods and gender division of labor with progressive income taxation. The tax income is spent on supplying public goods. The economic system consists of one production sector and one public sector. The public sector is financially supported by tax incomes. The model describes dynamic interactions of growth and gender with progressive income taxation. We simulated the model to demonstrate existence of equilibrium and motion of the dynamic system. We also examined the effects of changes in different parameters on the motion of the economic system. The model can be extended in multiple directions. For instance, it is straightforward to extend the model to multiple countries or/and multiple types of households. To focus on the role of progressive income taxation, this study only analyzes a homogeneous progressive taxation policy without taking account of possible taxation differences between gender and taxation on consumption and production. Our model is still limited to the neoclassical view of gender performance without taking account of institutional changes. Another important direction for future research is to explain the so-called glass ceiling effect. The term, introduced by Albrecht *et al.* (2003), refers to the phenomenon that the gender pay gap is increasing across the wage distribution and accelerating in the upper tail.

Appendix: Proving Lemma

From (1) and (11) we have

$$z \equiv \frac{r^* + \delta_k}{w} = \frac{N_i}{\beta_i K_i} = \frac{N_p}{\beta_p K_p}, \quad (\text{A1})$$

where $\bar{\beta}_j \equiv \alpha_j / \beta_j$, $j = i, p$. From (A1) and the specified form of the production function

$$F = A \bar{\beta}_i^{\beta_i} K_i z^{\beta_i}. \quad (\text{A2})$$

From (1) and (A2), we have

$$z = \left(\frac{r^* + \delta_k}{A \alpha_i} \right)^{1/\beta_i} \frac{1}{\bar{\beta}_i}, \quad w = \frac{A_w}{z^{\alpha_i}}, \quad (\text{A3})$$

where $A_w \equiv A \beta_i / \bar{\beta}_i^{\alpha_i}$. Insert (A3) in the definition of $\bar{y}$

$$\bar{y} = (1 + r^* \bar{\tau}_h) \bar{k} + \tilde{\alpha} \bar{\tau}_h, \quad (\text{A4})$$

where $\tilde{\alpha} \equiv (h_1 + h_2) w T_0$. Insert (7) in $T_j + \bar{T}_j = T_0$

$$T_j = \tilde{\alpha}_j - \frac{(1 + r^* \bar{\tau}_h) \tilde{\sigma}_j \bar{k}}{\bar{\tau}_h}, \quad (\text{A5})$$

where we use (A4) and

$$\tilde{\alpha}_j \equiv T_0 - \frac{\tilde{\alpha}}{h_j w}, \quad \tilde{\sigma}_j \equiv \frac{\sigma_j}{h_j w}.$$

From (A5) we have the labor force as

$$N = h - \frac{(1 + r^* \bar{\tau}_h) \tilde{\sigma} \bar{k}}{\bar{\tau}_h}, \quad (\text{A6})$$

where

$$h \equiv h_1 \tilde{\alpha}_1 + h_2 \tilde{\alpha}_2, \quad \tilde{\sigma} \equiv \tilde{\sigma}_1 + \tilde{\sigma}_2.$$

From (2) and (A5)

$$y_0 = r^* \bar{k} + \frac{w N}{N_0} = \left[r^* - \frac{(1 + r^* \bar{\tau}_h) w \tilde{\sigma}}{\bar{\tau}_h N_0} \right] \bar{k} + \frac{w h}{N_0}. \quad (\text{A7})$$

From (A7) and (3) we have

$$\bar{k} = \left[\left(\frac{\tau_h - \tau_0}{\tau_1} \right)^{1/a} - \frac{wh}{N_0} \left[r^* - \frac{(1 + r^* \bar{\tau}_h) w \tilde{\sigma}}{\bar{\tau}_h N_0} \right] \right]^{-1}. \quad (A8)$$

Insert (A1) in (12)

$$\bar{\beta}_i K_i + \bar{\beta}_p K_p = \frac{N}{z}. \quad (A9)$$

Insert $wN_p = \beta_p I_p$ in (9)

$$N_p = (r^* \bar{k} N_0 + wN) \frac{\beta_p \tau_h}{w}. \quad (A10)$$

We can see that the variables can be expressed as functions of τ_h as follows: N_p by (A5) $\rightarrow K_p$ by (A1) $\rightarrow I_p = wN_p / \beta_p \rightarrow z$ by (A8) $\rightarrow W$ by (A8) $\rightarrow \bar{k}$ by (A8) $\rightarrow \bar{K} = \bar{k} N_0$ $\rightarrow N$ by (A6) $\rightarrow K_i$ by (A9) $\rightarrow N_i$ by (A9) $\rightarrow y_0$ by (A7) $\rightarrow T_j$ by (A5) $\rightarrow \bar{T}_j$ by (A5) $\rightarrow \bar{y}$ by (A4) $\rightarrow F$ by (A2) $\rightarrow K$ by (A4) $\rightarrow G$ by the definition $\rightarrow \mathcal{C}$ and $\mathcal{S}$ by (A7) $\rightarrow w_j$ by (1). We express

$$\bar{k} = \Lambda(\tau_h). \quad (A11)$$

Taking derivatives of (A11) with respect to t yields

$$\dot{\bar{k}} = \frac{\partial \Lambda}{\partial \tau_h} \dot{\tau}_h. \quad (A12)$$

From (8) we have

$$\dot{\bar{k}} = \Lambda_0(\tau_h) \equiv s - \Lambda. \quad (A13)$$

From (A12) and (A13), we solve

$$\dot{\tau}_h = \bar{\Lambda}(\tau_h) \equiv \Lambda_0 \left(\frac{\partial \Lambda}{\partial \tau_h} \right)^{-1}. \quad (A14)$$

We thus proved the lemma.

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