Long Memory Volatility Models in R: Application to a *Regional Blue Chips Index*

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Abstract

An increasing number of researchers and analysts have formulated in the last decade innovative statistical equations for modelling and forecasting the volatility of financial returns. The evolution of technology and new software development have contributed positively to emerging economies, as well as in the improvement of market competition. We investigate in this study some experimental extensions of the multiplicative error model, which has been introduced by Engle (2002) for positive valued processes and it is specified as the product of a conditionally autoregressive scale factor and an innovation process with positive support. We use R 3.1.3 for the optimization, a modern software for statistical computing and graphics. Empirical analysis is carried out starting from May 2009 to January 2015. The equations fit fairly well the volatility of 'STOXX Balkan 50 Equal Weight' Index, which represents leading blue chips from eight Balkan countries in terms of free-float market cap. The models seem to absorb completely the slow decay of the global autocorrelation function.

Keywords: Stock Market Data, Volatility, MEM models, Balkan stock index.

1. Introduction

The volatility process is associated with the conditional variance of financial returns over time. It is a subject to be taken into consideration while assessing the risk of an investment in financial assets. A relatively high level of volatility means that the corresponding asset price tends to large variations over time, therefore the investor can record high profits or high losses. The realized volatility is a positive valued process with a highly persistent temporal dependency, indicated by a slow hyperbolic decay of the autocorrelation function. Granger and Joyeux (1980) and Hosking (1981) suggest the use of fractionally integrated autoregressive moving average (ARFIMA) model in order to capture the long term persistence in time series of many fields. Engle and Russell (1998) propose a different method for the analysis of data which arrive at irregular interval such as financial transaction series. The authors define a new statistical model called autoregressive conditional duration (ACD), which can be considered part of the generalized autoregressive conditional heteroskedasticity (GARCH) model, introduced by Bollerslev (1986). Engle (2002) extends the research suggesting the use of the multiplicative error model (MEM) for modelling non-negative processes. This model specifies an error that is multiplied times the conditional expectation that follows a GARCH dynamic. Brownlees, Cipollini and Gallo (2011) focus on the different possible specifications of the persistence in the conditional mean using applications on the volatility of components of the S&P100 index, following the logic of Engle and Lee (1999). Cipollini, Engle and Gallo (2013) suggest an extension to the vector MEM model defined by Cipollini, Engle and Gallo (2006), where the conditional expectation is given as a function of its past and the conditional expectations of other variables. The authors follow a semiparametric method lacking the necessity of specifying the probabilistic distribution function or the particular copula function. Barigozzi, Brownlees, Gallo and Veredas (2014) expand the MEM model introduced by Engle (2002), decomposing the conditional expectation as the product of a systematic trend and an idiosyncratic dynamic component. This model, named SeminonParametric Vector MEM (SPvMEM), can be used to estimate panels of market activity, risk and liquidity measures.

We suggest in this paper an interesting flexible MEM specification in which the process for realized volatility can be seen as a mixture of two Gamma MEMs with different coefficients for the conditional expectation and different shape parameters for the Gamma (De Luca and Gallo, 2007; Gallo and Otranto, 2014). We extend this framework even further adopting a mixed Gamma model for a MEM with fixed parameters which depend on the value of the past (Lanne, 2006; Ahoniemi and Lanne, 2013).

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The paper is organized as follows. Section 2 establishes the notation by summarizing some of the features of the multiplicative error model. Section 3 contains features of the equations and some empirical results illustrating the behaviour of the two MEM specifications for the realized volatility of 'STOXX Balkan 50 Equal Weight' Index. Section 4 concludes.

2. Theoretical framework

Let us consider r_t to be the daily log return at time t with negligible mean. We use a common indicator of volatility y_t, the daily squared return. The general structure of a multiplicative error model was outlined by Engle (2002) for realized volatility. A linear *MEM(p,q)* is given by:

$$y_t = \mu_t \varepsilon_t$$

where, conditionally to information I_{t-1}: μ_t is a non-negative predictable process. The μ_t evolution depends on a vector of unknown parameters θ, $\mu_t = \mu_t(\vartheta)$. ε_t follows a distribution with nonnegative support, with mean 1 and unknown variance σ^2. We have:

$$\varepsilon_t | I_{t-1} \sim D(1, \sigma^2).$$

These definitions provide us:

$$E(y_t | I_{t-1}) = \mu_t$$

$$V(y_t | I_{t-1}) = \sigma^2 \mu_t^2$$

In order to close the model, we adopt a density function for ε_t and specify an equation for the conditional mean μ_t. A common choice in many studies, is to use the exponential distribution, a particular case of the Gamma distribution. We have:

$$\varepsilon_t | I_{t-1} \sim Gamma(\phi, 1/\phi),$$

where ϕ >0, $E(\varepsilon_t | I_{t-1}) = 1$ and $V(\varepsilon_t | I_{t-1}) = 1/\phi$. The density function conditioned to the innovation ε_t is:

$$f(\varepsilon_t | I_{t-1}) = \frac{1}{\Gamma(\phi)} \phi^\phi \varepsilon_t^{\phi-1} \exp\{-\phi \varepsilon_t\}.$$

We can conclude that y_t follows a Gamma conditioned distribution

$$y_t | I_{t-1} \sim Gamma(\phi, \mu_t / \phi)$$

with density function:

$$f(y_t | I_{t-1}) = \frac{1}{\Gamma(\phi)} \frac{\phi^\phi}{\mu_t^\phi} y_t^{\phi-1} \exp\{-\phi \cdot y_t / \mu_t\}.$$

So, we have in this case:

$$E(y_t | I_{t-1}) = \phi \cdot \frac{\mu_t}{\phi} = \mu_t$$

$$V(y_t | I_{t-1}) = \phi \cdot \frac{\mu_t^2}{\phi^2} = \frac{\mu_t^2}{\phi}.$$

With regards to μ_t, we consider the simplest specification, the GARCH(p,q).

$$\mu_t = \omega + \sum_{i=1}^{p} \alpha \cdot y_{t-i} + \sum_{j=1}^{q} \beta_j \cdot \mu_{t-j} \ .$$

We estimate the parameters included in μ_t , let's call them θ. The log-likelihood function useful for the estimation is:

$$\log L(y_t ; \theta) = l(y_t ; \theta) = c - \phi \sum_{t=1}^{T} \left(\log \mu_t + \frac{y_t}{\mu_t} \right),$$

where c is an irrelevant constant to the estimation method depending on Φ. We maximize the function with respect to θ:

$$\sum_{t=1}^{T} \left(\frac{x_t - \mu_t}{\mu_t^2} \right) \frac{\partial \mu_t}{\partial \theta} = 0 \ .$$

We obtain the our estimators $\hat{\theta}_{SMV}$ using iterative procedures. We have:

$$\hat{\theta}_{SMV} = \arg\max_\theta l(y_t ; \theta)$$

The asymptotic distribution of the estimators under regularity conditions will be:

$$\sqrt{T} (\hat{\theta}_n - \theta)^{asy} \sim N(0, V_{as})$$

$$\hat{\theta}_n^{\ asy} \sim N(\theta, \frac{1}{T} V_{as}),$$

where $V_{as} = J^{-1}$ is the asymptotic variance. We have:

$$J = E\left[-\frac{\partial^2 l(y_t ; \theta)}{\partial \theta \partial \theta'} \right] ,$$

$$\hat{J} = -\frac{1}{T} \sum_{i=1}^{T} \frac{\partial^2 l(y_t ; \theta)}{\partial \theta \partial \theta'} = \frac{1}{T} \sum_{i=1}^{T} \frac{\partial l(y_t ; \theta)}{\partial \theta} \frac{\partial l(y_t ; \theta)}{\partial \theta'} \Big|_{\theta = \theta'}$$

If order to verify the null hypothesis $H_0 : \theta_i = \theta_{i,0}$, we use the Student's t test:

$$t = \frac{\hat{\theta}_i - \theta_{i,0}}{\sqrt{V_{ii}}} ,$$

where $\hat{\theta}_i$ is the *i*-th row vector and V_{ii} is the *(i,i)* element of the $\frac{1}{T}\hat{V}_{as} = \frac{1}{T}\hat{J}^{-1}$ matrix.

3. MEM equations and application

Initially, we consider an interesting formulation of MEM models (Lanne, 2006; Ahoniemi and Lanne, 2013), where the innovation term ε_t follows a mixed gamma distribution with constant parameters. The innovation mean is unitary

respecting the establishment of the previous section. Let's call y_t the realized volatility of a generic stock. We have:

$$y_t = \mu_t \varepsilon_t .$$

The innovation is defined as follows:

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$$\varepsilon_t = \begin{cases} \varepsilon_{1,t} \; with \; prob. \quad \pi \\ \varepsilon_{2,t} \; with \; prob.(1-\pi) \end{cases},$$

where $0 < \pi < 1$ and

$$\varepsilon_{1,t} \sim Gamma(\gamma_1, 1/\gamma_1)$$

$$\varepsilon_{2,t} \sim Gamma(\gamma_2, 1/\gamma_2).$$

The general form of the conditional mean equation is:

$$\mu_{1t} = \omega_1 + \sum_{i=1}^{q} \alpha_{1i} \cdot y_{t-i} + \sum_{j=1}^{p} \beta_{1j} \cdot \mu_{1,t-j}$$

$$\mu_{2t} = \omega_2 + \sum_{i=1}^{q} \alpha_{2i} \cdot y_{t-i} + \sum_{j=1}^{p} \beta_{2j} \cdot \mu_{2,t-j}.$$

We consider also a model, belonging to the MEM family, with an innovation term distributed according to a mixture of Gamma with variable weights (De Luca and Gallo , 2007; Gallo and Otranto, 2014). The overall mean of the innovation term is always unitary. So, we have:

$$\varepsilon_t = \begin{cases} \varepsilon_{1,t} \; with \; prob. \quad \pi_t \\ \varepsilon_{2,t} \; with \; prob.(1-\pi_t) \end{cases}$$

$$\varepsilon_{1,t} \sim Gamma(\lambda_1, \gamma_1)$$

$$\varepsilon_{2,t} \sim Gamma(\lambda_2, \gamma_{2,t})$$

where $0 < \pi < 1$. We suppose that weights depend on their past and on the previous value of realized volatility:

$$\pi_t = \frac{\exp\{\delta_0 + \delta_1 \cdot y_{t-1} + \delta_2 \pi_{t-1}\}}{1 + \exp\{\delta_0 + \delta_1 \cdot y_{t-1} + \delta_2 \pi_{t-1}\}}$$

Imposing the unitary mean constraint, we have:

$$\gamma_{2,t} = \frac{1 - \pi_t \lambda_1 \gamma_1}{(1-\pi_t)\lambda_2}.$$

We call the two models respectively, LANNE and DLG in our analysis. After defining the regression models, it is important to confirm the goodness of fit of the model and the statistical significance of the estimated parameters. In simple linear models the errors follow a normal distribution, but in some regressions based on mixture of distributions this hypothesis may not be valid. The method used in this paper is based on the so-called probability integral transformation of Rosenblatt (1952), which is a universally approach of transforming a set of dependent variables into independent uniform distributed variables. Dunn and Smyth (1996) define and apply the quantile residuals to regression models with independent responses.

Let $F(y; \mu, \phi)$ be the cumulative distribution function of $P(\mu, \phi)$. If F is continuous, then the $F(y_i; \mu, \phi)$ are uniformly distributed on the unit interval. The quantile residuals are defined by

$$r_{q,i} = \Phi^{-1}\left\{F(y_i; \hat{\mu}_i, \hat{\phi})\right\}$$

where $\Phi()$ is the cumulative distribution function of the standard normal. Quantile residuals are independent and follow a standard normal distribution if the model is correctly specified and the parameters are consistently estimated.

We start with considering the realized volatility of *'STOXX Balkan 50 Equal Weight' Index*, which represents the largest and most liquid companies across eight Balkan countries. The index covers 50 stocks from Bulgaria, Croatia, Greece, the Former Yugoslav Republic of Macedonia (FYROM), Romania, Serbia, Slovenia and Turkey. We consider the period from 1st May 2009 to 30th January 2015. The time series of daily realized volatility has been built as the square root of realized variance. Figures 1 displays the plot of realized volatility. Figure 2 shows the estimated global autocorrelation function and partial autocorrelation function of realized volatility which exhibit a very slow decay, that is a long-memory pattern.
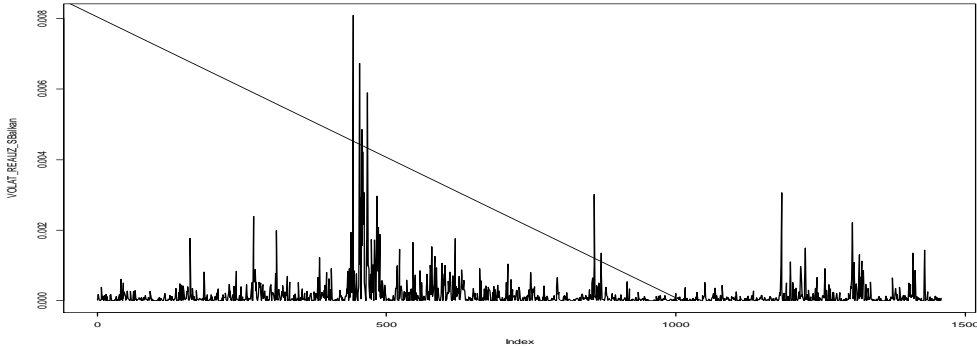

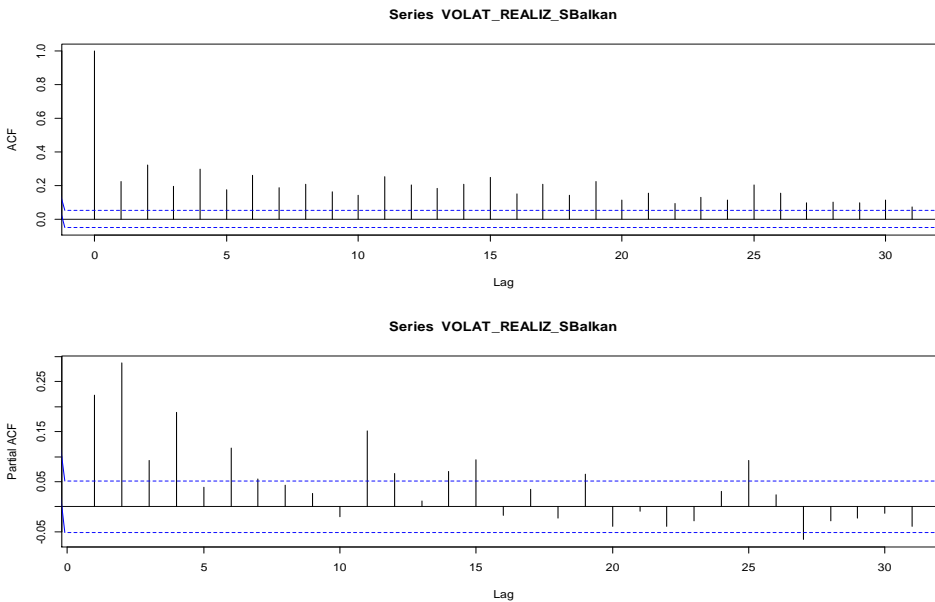**Figure 1. Time series of *'STOXX Balkan 50 Equal Weight'* realized volatility.**

Figure 2. Global Autocorrelation Function and Partial Autocorrelation Function for '*STOXX Balkan 50 Equal Weight'* realized volatility.

We try to model the realized volatility series with both possible specifications of MEM models. Let's see what happens with the LANNE specification. This is the conditional mean equation, where p = 2 and q = 1:

$$\mu_t = \omega + \alpha_1 \cdot y_{t-1} + \alpha_2 \cdot y_{t-2} + \beta_1 \mu_{t-1}.$$

We can see in the tables below the estimate, the standard error, the statistic test value and the observed p-value for each of the seven parameters. We simply observe the last column of the table where is calculated the p-value for the statistical test. We can fix a priori a p-value of 5% for acceptance of the null hypothesis. So, we accept 'H0: The parameter is null' if the observed p-value is greater than 5% and vice versa if it is less than 5%. As we can see from the table we refuse the null hypothesis in all cases, a positive indicator for the model evaluation.

Parameter	Estimation	p-value
π	0.067	~0
γ_1	5.634	~0
ω_1	0.127	~0
α_{11}	0.374	~0
β_{11}	0.749	~0
γ_2	11.564	~0
ω_2	0.011	~0
α_{21}	0.378	~0
β_{21}	0.769	~0

Table 1. Parameters test, LANNE model.

Let's check now if the model is specified correctly or not using the quantile residuals method mentioned above. We have represented in figure 3 (from the left to the right) the quantile residual graph, the quantile-quantile graph (if the distribution approximates the normal standard one), the global autocorrelation (ACF) and the partial autocorrelation (PACF) of the quantile residuals and of the squared quantile residuals for LANNE model. We observe that the approximation is generally good.
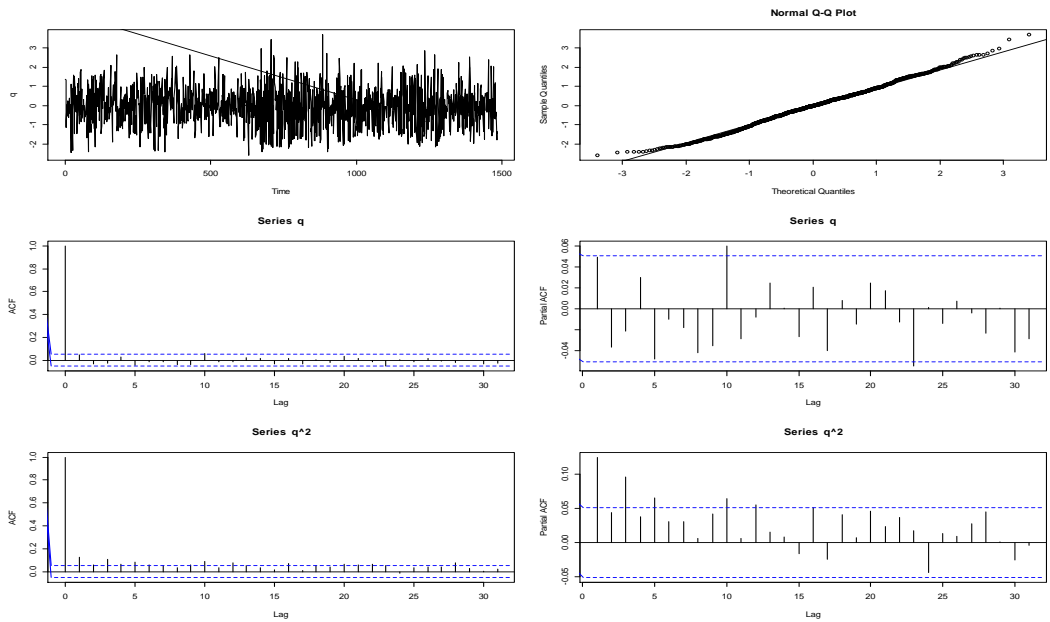
Figure 3. Quantile residuals graphs, LANNE model.

Let's try now to estimate the our time series through the other MEM formulation, the DLG one. This specification is a mixture of two Gamma MEMs with different coefficients for the conditional expectation. The equation for the conditional mean will be:

$$\mu_t = \omega + \alpha_1 \cdot y_{t-1} + \alpha_2 \cdot y_{t-2} + \beta_1 \mu_{t-1}$$

We use the method of Maximum Likelihood (ML) to estimate the parameters of interest.

Parametro	Stima	p-value
λ_1	6.723	~0
λ_2	16.016	~0
ω	0.099	~0
α_1	0.374	~0
α_2	0.081	0.805
β_1	0.631	~0
δ_0	0.905	~0
δ_1	0.983	~0
δ_2	17.106	~0
γ_1	0.419	~0

Table 2. Parameters test, DLG model.

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As we can see from the table, we refuse the H0 hypothesis in all cases, except for α_2. The p-value in this case is greater than 5%, so we accept the null hypothesis that the coefficient is statistically equal to zero. We use even in this case the quantile residuals method. We check if residuals are independent and approximate the standard normal distribution. If this is true, the model is correctly specified and the parameters are consistently estimated. If not, quantile residuals are expected to exhibit different characteristics, i.e. dependence, autocorrelation or different distribution.
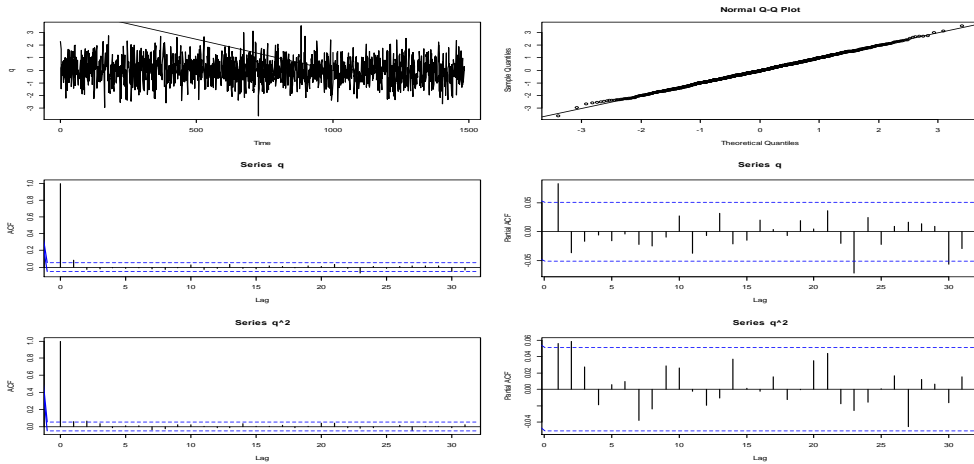

Figure 4. Quantile residuals graphs, DLG model.

We have represented in figure 4 the quantile residual graph, the quantile-quantile graph (if the distribution approximates the normal standard one), the global autocorrelation (ACF) and the partial autocorrelation (PACF) of the quantile residuals and of the squared quantile residuals for the estimated model. We observe that the points line up perfectly to the normal distribution. The global and partial autocorrelations are, in most cases, within the confidence bands. We conclude that the model is correctly specified and the estimates are consistent.

Now we compare our models using well known selection criteria as AIC (Asymptotic Information Criterion; Akaike, 1974) and BIC (Bayesian Information Criterion; Schwarz, 1978). The first criterion, AIC, is defined as:

$$AIC(k) = 2 \cdot k - 2 \cdot l_T ,$$

where l_T is the maximized value of the likelihood function for the estimated model, *k* is the number of parameters in the statistical model and T is the number of data points. BIC is defined as:

$$BIC(k) = k \cdot \log(T) - 2 \cdot l_T .$$

We have presented in the table below the maximized value of the likelihood function, AIC and BIC for the estimated models.

MEM Model	lt	AIC	BIC
LANNE	-109,217	236,434	281,704
DLG	-71,638	163,276	213,576

Table 3. Maximized value of the likelihood function, AIC and BIC criteria.

Lower values of AIC and BIC, as well as higher values of the maximized value of the likelihood function correspond to the most fitting model and vice versa. In this case, we can conclude that DLG model fits better the corresponding data sample.

4. Concluding remarks

We consider in this study two possible innovative equations for the realized volatility of financial returns, part of the multiplicative error models family, called LANNE and DLG respectively. We estimate and compare these models using some applications on 'STOXX Balkan 50 Equal Weight' Index, for the period from 1 May 2009 to 30 January 2015. The error term of LANNE equation follows a gamma distribution with constant parameters, whereas the error term of DLG equation is distributed according to a mixture of Gamma with variable weights.

We compute the quantile residuals in R 3.1.3 software, in order to measure how well the specific MEM models fit the corresponding data sample. We observe positive signals of independence and a satisfactory approximation of residuals to the standard normal distribution. This implies that the equations fit fairly well the volatility of 'STOXX Balkan 50 Equal Weight' Index, and the corresponding coefficients are consistently estimated. We also calculate the maximized value of the likelihood function, and the AIC and BIC criteria for each model, concluding that DLG model fits better the data. This study can be enriched with supplemental analysis regarding other possible approaches of volatility estimation and other possible variables. Furthermore, we can test and compare these methods through the quantile residuals analysis, as well as through forecasts.

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