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Optical Solitons and Demonstration of Its Application

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Abstract

Solitons are structurally stable solitary waves that propagate in a nonlinear medium. In this paper, solitons will be considered as the basis for solving many classical nonlinear equations of motion. Some classical solutions that were modeled through the application of Wolfram Mathematica System and MATLAB programming language. In this paper some soliton solutions will also be compared and some types of solitons were modeled.The dynamics of solitons was studied in consideration of solutions of some equations, such as the Korteweg - de Vries equation and as a particular solution for the nonlinear Schrödinger equation provided that the nonlinearity parameter R>0 in the equation. We concluded by showing solitons in more detail which are often used in practice as a simpler method for explaining complex phenomena and solving non-classical equations

Keywords: Soliton, Shrodinger non-linear equation, Korteweg–de Vries equation, optical soliton, soliton simulations.

Introduction

At present, the theory of solitons has embraced various branches of the natural sciences. Initially, they arose in the study of waves in water and in other problems of hydrodynamics [1, 2]. Afterwards, the solitons penetrated together with the hydrodynamic model into plasma physics and condensed matter physics. Later solitons and phenomena associated with them began to be studied in classical and quantum field theory and statistical mechanics. Solitons are also found in such areas as biophysics, nonlinear optics, etc. It must be emphasized that the study of solitons in nonlinear optics has been possible both theoretically and experimentally.

Most of the considered waves are in the group of monochromatic waves. But besides them, there is a wide group of waves, which are called solitary [1]. A good example of such a wave is the light pulse. Very often, a solitary wave is presented in the form of a wave packet, i.e., a linear superposition of a large number of monochromatic waves having frequencies close to the frequency of the carrier wave. Often, each of the components of a wave packet in space propagates with its own speed, i.e. there is a

velocity dispersion. This phenomenon leads to an increase in the width of the wave packet, i. e. the broadening of its dispersion [5]. The speed of the entire packet is called the group velocity, and the mediums in which the velocity dispersion is present are called dispersing.

In 1965, N. Zabuski and M. Kruskal discovered that solutions of the Korteweg – de Vries equation describing the propagation of solitary waves in shallow water have remarkable properties: they do not experience dispersive broadening and interact elastically, i.e. they retain their shape after collision and passage through each other [1, 2]. To emphasize the exceptional elementary nature of these solitary waves, they were given the name "soliton". Solitons are defined as follows: this is a special type of nonlinear solitary waves (wave packets) that retain their shape and speed during their own motion and collisions with each other [1].

As we know, intense high-frequency (HF) wave packets can propagate in nonlinear mediums without changing their shape, i.e. they are solitons. Soliton solutions arise in many topical problems in various fields of physics while modeling the propagation of intense waves in dispersive media. In physics, there are many types of solitons, such as dark solitons, light solitons, optical solitons, etc.

Different types of solitons are particular solutions of many equations, such as the Korteweg – de Vries equation, the nonlinear Schrödinger equation with the condition that the nonlinearity parameter in the equation is $R > 0$, the Maxwell – Bloch system [4], the sine – Gordon equation, and so on.

In this paper, we will consider some particular solutions of the above equations. These solutions can be presented in the following forms: one-soliton solutions, two-soliton solutions, and cases where the solution is optical solitons. All solutions are modeled and presented in the figures below.

Types of solitons and their applications

As we mentioned above, solitons are solutions of many equations in physics [1]. The below-discussed solitons are among the more widely used solutions of equations. In addition, they are o within the research scope of various fields of physics.

Dark solitons

As we already understood, a soliton is a wave traveling in a nonlinear medium by itself. A dark soliton is formed when this intensity locally decreases in a continuous wave of certain intensity[1, 5]. In other words, these are gaps in the wave, no matter how rough it sounds (fig. 1).

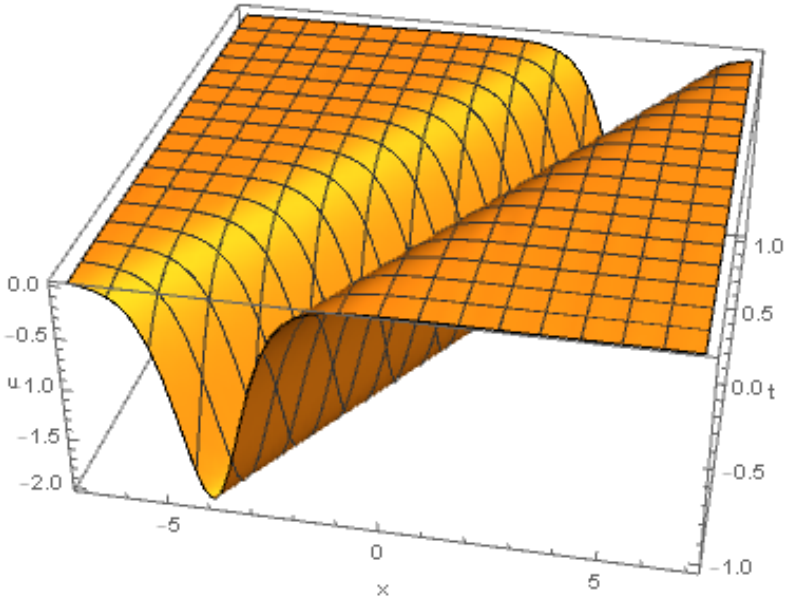

Fig. 1. Dark Soliton

The frequency crests of the microcavity use the nonlinear Kerr effect in the integrated optical cavity to generate a variety of phase-frequency lines [1]. The interval between the lines can reach 100 GHz, which makes the system an excellent multi-wavelength light source for fiber-optic devices and systems. The dispersion of the microresonator affects the physical dynamics itself. Recent studies of the states of the frequency crest have demonstrated the formation of dark pulses in a microcavity with normal dispersion. This kind of "dark-impulse" ridges have become very popular among researchers because of their possible use in coherent communications due to the very high efficiency.

Optical solitons

Optical solitons are optical pulses that preserve the structural stability of the envelope when propagating in a nonlinear medium even in the presence of interfering factors and interactions with other solitons [2, 3]. Depending on the nature of the nonlinear interaction of radiation with matter, the soliton effects in optics are divided into resonant and non-resonant. In non-resonance media, optical solitons are formed as a result of the balance of two competing processes — dispersive spreading and nonlinear self-compression. The most favourable conditions for the formation of a soliton are realized in single-mode optical fibres due to extremely small optical losses and stability of the mode structure of the radiation with an increase in input power up to values close to the self-focusing threshold.

The basis for an adequate mathematical description of the processes of formation and interaction of solitons in the picosecond range of durations is the nonlinear Schrödinger

equation, which corresponds to the complex amplitude of the field $A(\xi, \tau)$ [1]. The envelope of a soliton pulse has the form $A(\xi,\tau) = \mathrm{sech}(\tau)e^{-i\xi/2}$, where ξ - is the distance normalized to the dispersion length L_D, $\tau = (1 - z/u)/\tau_0^2 -$ is the running time normalized to the initial pulse duration, u is the group velocity. Schrödinger nonlinear equation belongs to the class of integral nonlinear equations and is solved by the inverse scattering problem. If the power of a spectrally bounded pulse exceeds the critical power, then its asymptotic behaviour as $\xi \to \infty$ is determined by the soliton component. The amplitude of the non-soliton part of the solution decreases.

An important factor in the analytically calculated solutions of a nonlinear Schrödinger equation is N-soliton pulses corresponding to initial conditions of the form $A(0,\tau) = N\mathrm{sech}(\tau)$, where N is an integer. They are a nonlinear superposition of N moving with the same speed solitons with amplitudes $A_m = (2b-1)$, $b = 1, 2, \ldots N$. Important features of N-soliton pulses are that their propagation begins with self-compression, and the complex amplitude modulus is periodic in ξ with a period $\pi/2$.

Fundamental soliton

As we have already mentioned, the bandwidth of fibre-optic communication lines is limited to non-linear effects and dispersion, changing the amplitude of the signals and their frequency [1]. But, on the other hand, the same nonlinearity and dispersion can lead to the creation of solitons, which retain their shape and other parameters substantially longer than anything else. An example of a laser that changes the refractive index inside an optical fiber as it spreads is vital enough, especially if a pulse of several watts is placed into a fiber thinner than a human hair. For comparison, we will clarify that a typical 9-watt energy-saving light bulb illuminates a desk, but is palm-sized at the same time. In general, we will not be far from reality assuming that the dependence of the refractive index on the pulse power inside the fiber will look as follows (1):

$$n(P) = n_0 + n_1 P, \quad n_1 > 0 \qquad (1)$$

After physical reflections and mathematical transformations of varying complexity of amplitude a of the electric field inside the fiber, one will get the equation of the form (2)

$$\frac{1}{2}\frac{\partial^2 a}{\partial x^2} + i\frac{\partial a}{\partial z} - N^2|a|^2 a = 0 \qquad (2)$$

where z and x coordinate along the propagation of the beam and transverse to it. The N coefficient plays an important role. It determines the relationship between dispersion and nonlinearity. If it is very small, then the last term in the formula can be thrown out due to the weakness of the nonlinearities. If the coefficient is very large, then the

nonlinearity, pressing on the dispersion, will single-handedly determine the features of signal propagation. So far, they tried to solve this equation only for integer values of N. So, for N = 1, the result is especially simple (3) :

$$a(x,z) = \operatorname{sech}(x)\exp(-iz/2) \qquad (3)$$

The function of the hyperbolic secant looks like an ordinary "bell"

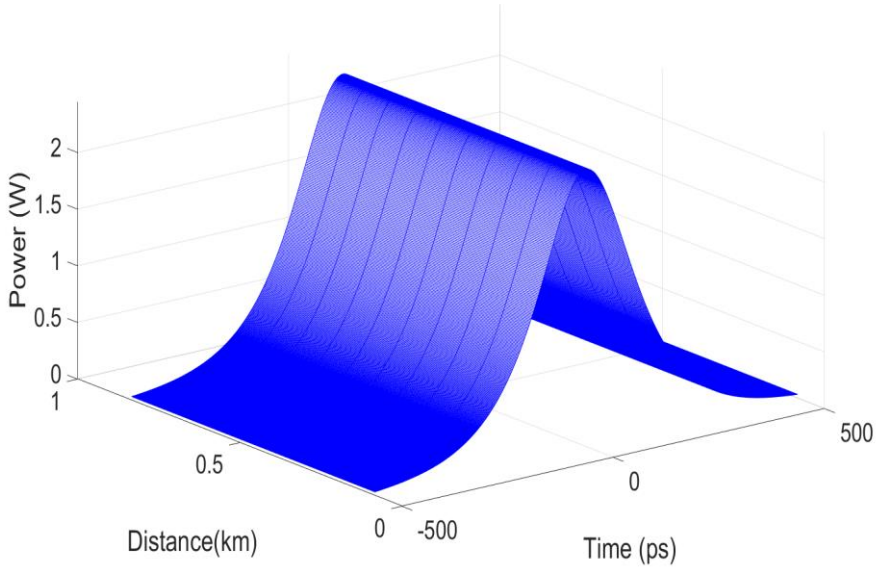

Fig.2. The intensity distribution in the cross section of the laser beam in the form of a fundamental soliton

and is called the fundamental soliton (fig. 2). The imaginary exponent determines the soliton distribution along the fiber axis. In practice, this all means that having shone on the wall, we would see a bright spot in the center, the intensity of which would quickly fall off at the edges.

The fundamental soliton, like all solitons arising using lasers, has certain features. First, if the laser power is insufficient, it will not appear. Secondly, even if somewhere the fiber bends, the soliton passing through the damaged area will change, but will quickly return to its original parameters [1]. People and other living beings also fall under the definition of an autosoliton (the ability to return to a quiet state), which is important in nature.

Second order soliton

The Korteweg-de Vries Equation (KdV equation) describes the theory of water waves in shallow channels, such as a canal [1, 6]. It is a non-linear equation which exhibits special solutions, known as solitons, which are stable and do not disperse with time.

Furthermore there as solutions with more than one soliton which can move towards each other, interact and then emerge at the same speed with no change in shape (but with a time "lag" or "speed up").

The form of Korteweg-de Vries Equation is shown below (4).

$$\frac{\partial u}{\partial t} = 6u\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial x^3} \qquad (4)$$

The theory for solutions with more than one soliton is complicated and we will not discuss it, but rather just display a two-soliton solution, verify that it is indeed a solution, and look at its properties [6]. Specifying adequate resolution and number of time steps, my computer ran out of memory.

The theory states that an initial state (5)

$$u(x,0) = -n(n+1)\,\mathrm{sech}^2(x) \quad (5)$$

results in n solitons that propagate with different velocities. The solution for n = 2 is (6)

$$u(x,t) = -12\frac{3+4\cosh(2x-8t)+\cosh(4x-64t)}{\left[3\cosh(x-28t)+\cosh(3x-36t)\right]^2} \qquad (6)$$

It is not immediately evident that the above expression for u(x, t) satisfies the KdV equation, but Mathematica confirms that it does:

Next we plot the solution at time t = 1 in fig. 3:

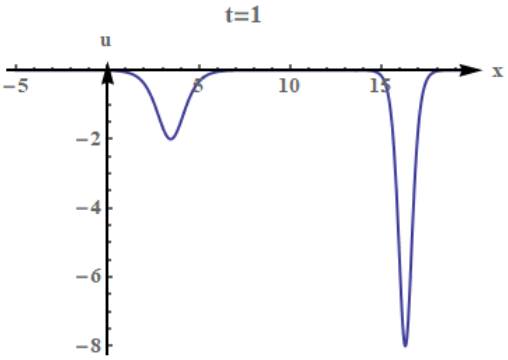

Fig. 3 KdV equation two soliton solution at $t=1$

Other two soliton solution example is sine-Gordon equation [7, 8]. The sine-Gordon equation is a nonlinear hyperbolic partial differential equation in $1 + 1$ dimensions involving the d'Alembert operator and the sine of the unknown function. There are two equivalent forms of the sine-Gordon equation. In the (real) space-time coordinates, denoted (x, t), the equation reads (7):

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = \sin u \qquad (7)$$

where partial derivatives are denoted by subscripts. Passing to the light cone coordinates (u, v), akin to asymptotic coordinates where (8)

$$f = \frac{x+t}{2}, \quad v = \frac{x-t}{2} \qquad (8)$$

the equation takes the form (9):

$$\frac{\partial^2 u}{\partial f\, \partial v} = \sin(u) \qquad (9)$$

Multi-soliton solutions can be obtained through continued application of the Bäcklund transform to the 1-soliton solution, as prescribed by a Bianchi lattice relating the transformed results. The 2-soliton solutions of the sine-Gordon equation show some of the characteristic features of the solitons. The traveling sine-Gordon kinks and/or antikinks pass through each other as if perfectly permeable, and the only observed effect is a phase shift [5, 7]. Since the colliding solitons recover their velocity and shape such kind of interaction is called an elastic collision (fig. 4).

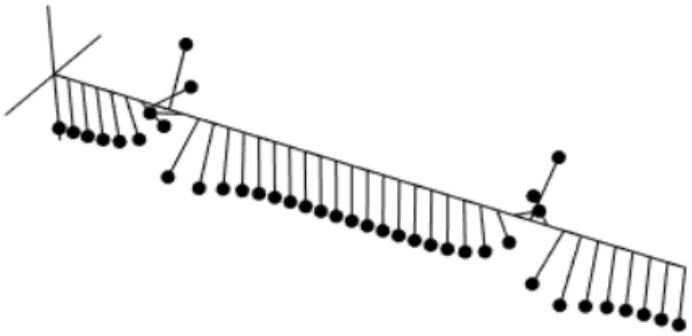

Fig.4 Two solitons Kink-kink collision

Third order soliton

For sine-Gordon equation has three soliton solution. 3-soliton collisions between a traveling kink and a standing breather or a traveling antikink and a standing breather results in a phase shift of the standing breather. In the process of collision between a moving kink and a standing breather, the shift of the breather Δ_B is given by (10) [5]:

$$\Delta_B = \frac{2\text{arctanh}(\sqrt{(1-\omega^2)(1-v_k^2)}}{\sqrt{1-\omega^2}} \qquad (10)$$

where v_k is the velocity of the kink, and ω is the breather's frequency. If the old position of the standing breather is x_0, after the collision the new position will be $x_0 + \Delta_B$ (fig. 5)[8].

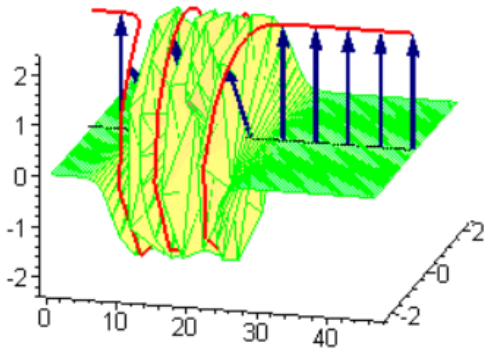

Fig. 5 Moving kink standing breather collision

Conclusion

In this paper, we considered different types of solitons as the basis for solving some nonlinear equations. Particular solutions of the following equations were presented: the non-linear Schrödinger equation, the sine-Gordon equation, and the Korteweg-de Vries equation. Monosoliton, two-soliton and three-soliton solutions were shown. In addition, the influence of the dark soliton on the wave and its significance in modern literature was shown.

Using computer simulation, the behaviour of solitons in a nonlinear and dispersive medium was shown with a particular one-soliton solution of the Schrödinger equation.

In addition to the above, the behaviour of a laser beam in the form of a fundamental soliton was modelled. In this paper, it was proved that solitons are one of the easiest ways to explain complex phenomena and solve non-classical equations.

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